**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. **0.2676 - Correct**
4. 0.5
5. 0.6987

**Ans:**

**Mean = 45**

**Standard Deviation = 8**

**Let X be the time taken to complete servicing.**

**Pr (X>50) = 1 – Pr (X<= 50)**

**1-stats.norm.cdf(50,45,8).round(4)**

**0.2676**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.

**Ans:**

**Mean = 38**

**Standard Deviation = 6**

**N = 400**

1. More employees at the processing center are older than 44 than between 38 and 44. - **FALSE**

**Pr(X>44) = 1-Pr(X<=44)**

**1-stats.norm.cdf(44,38,6).round(4)**

**0.1586**

**Probability of employees older than 44 is 15.86% i.e., around 63 employees.**

**Probability of employees less than 44 is 84.13%**

**stats.norm.cdf(44,38,6).round(4)**

**0.8413**

**Pr(X>38) = 1-Pr(X<=38)**

**1-stats.norm.cdf(38,38,6)**

**0.5**

**Probability of employees between 38 and 44 is 0.8413 - 0.50 = 0.3413 i.e., 34.13% i.e., around 136 employees**

**Hence the given statement is FALSE**

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees. - **TRUE**

**Probability of employees under the age of 30 is 9.12 i.e., around 36 employees, hence the above statement is TRUE.**

**stats.norm.cdf(30,38,6)**

**0.0912**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans:**

**2X1 is double the size of X1, so 2X1 is also normally distributed.**

**Since X1 and X2 are normal, X1+X2 are normal, since the sum and random samples of normal distributions are normal with appropriate parameters.**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. **48.5, 151.5 Correct**
6. 90.1, 109.9

**Ans**: **Since we need to find out the values of a and b, which are symmetric about the mean, such that the probability of random variable taking a value between them is 0.99, we have to work out in reverse order.**

**The Probability of getting value between a and b should be 0.99**

**The Probability outside the a and b area is 0.01 i.e., 1-0.99**

**The Probability towards left from a = -0.005 i.e., 0.01/2**

**The Probability towards right from b = +0.005 i.e., 0.01/2**

**So since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.**

**For Probability 0.005 the Z Value is -2.57 (from Z Table).**

**Z(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4**

**Z(+0.005)\*20+100 = (-2.57)\*20+100 = 48.6**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans:**

**Let X be the sum of two distributions,**

**E[X] = E[45\*(profit1+profit2)] = 45\*(5+7)=540 Million Rs**

**SD[X] = SD[profit1+profit2] = 45\*√(9+16) = 225 Million Rs.**

**X ~ (540,2252)**

1. **As per empirical rule, 95% of profitability lies = (540-450,540+450) = (90,990)**

**B. 5th percentile from left side = 540-1.5\*225 = 202.5 Million Rs**

**C. For 1st division, p-value = stats.norm.cdf(0,5,3) = 0.047**

**For 2nd division, p-value = stats.norm.cdf(0,7,4) = 0.040**

**Since p-value for 2nd division is lower, 2nd division has large probability of making loss.**